**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987
6. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
7. More employees at the processing center are older than 44 than between 38 and 44.
8. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

A. True.

- Because the average age is 38 and the standard deviation is 6, we can use these values to understand the distribution.

- About 68% of employees fall within one standard deviation from the mean.

- Since 44 is one standard deviation above the mean, we can expect approximately 68% of employees to be older than 44.

- However, between 38 and 44 is within one standard deviation from the mean, so there's another 68% there.

- Hence, more employees are likely to be older than 44 than between 38 and 44.

B. False.

- To find the number of employees under 30, we need to use the mean and standard deviation.

- Using these values, we find that the proportion of employees under 30 is about 9.86%.

- If we multiply this by the total number of employees (400), we expect around 39 employees under 30, not 36.

- Therefore, the statement is False.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Both 2x1 and 2 x square represent combinations of random variables x1 and x2, which are independent and identically distributed (iid) normal random variables.

1. For 2x1:

- Doubling x1 leads to another normal distribution, denoted as 2x1, with its mean doubled 2mu and its variance quadrupled 4\sigma^2.

- This implies that the values of 2x1 are more spread out compared to the values of X1, resulting in a broader distribution.

2. For X1 + X2:

- Adding both results in another normal distribution, denoted as (X1 + X2), with its mean being the sum of the individual means 2mu and its variance being the sum of the individual variances 2\sigma^2.

- This implies that the values of (X1 + X2) are less spread out compared to the values of 2X1, resulting in a narrower distribution.

In essence, both 2X1 and X1 + X2 share the same mean (twice the mean of X1), but they differ in their spread of values, as indicated by their variances. While 2X1 has a higher variance and thus a wider spread of values, (X1 + X2 has a lower variance and a narrower spread of values.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9
7. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
8. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
9. Specify the 5th percentile of profit (in Rupees) for the company
10. Which of the two divisions has a larger probability of making a loss in a given year?

A. To find the 95% probability range for the total annual profit in rupees, we calculate the mean and standard deviation of the combined profit distribution. The mean of the total profit is 225 + 315 = 540 million rupees, and the standard deviation is √((135)^2 + (180)^2) ≈ 224 million rupees. Using the 95% confidence interval formula for a normal distribution, we find the range to be approximately 540 ± 1.96 \* 224 million rupees, or about 99 to 981 million rupees.

B. The 5th percentile of the total profit distribution in rupees is found by using the inverse cumulative distribution function (CDF) of the normal distribution. This corresponds to the value below which 5% of profits fall. Converting from dollars to rupees, we find the 5th percentile to be approximately 99 million rupees.

C. To determine which division has a higher probability of making a loss, we calculate the probability of each division's profit being below zero using the standard normal distribution. The probability of Profit1 being negative is approximately 0.0013, while the probability of Profit2 being negative is approximately 0.00003. Therefore, Profit2 has a smaller probability of making a loss compared to Profit1.